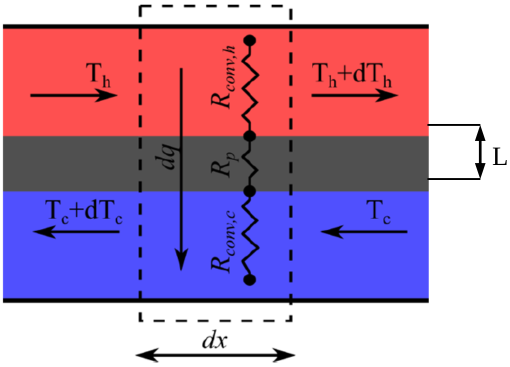
# Design of heat exchangers

## Overall heat transfer coefficient

Consider energy balance in a differential segment of a single-pass heat exchanger shown schematically in figure. The rate of heat transfer in this segment is

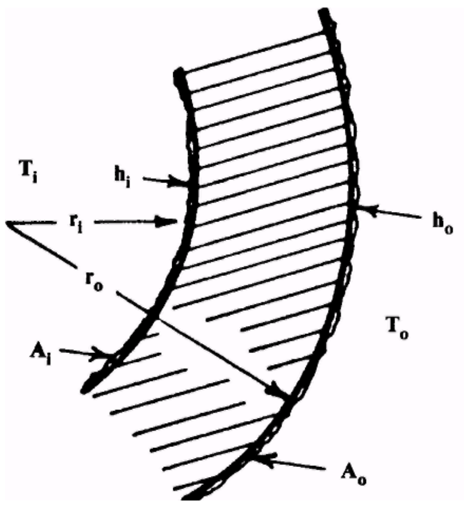
$$dq(x) = U\Delta T(x)dA(x),$$




where $U$ is the overall heat transfer coefficient, $\Delta T$ is the local temperature difference between the hot and cold fluids, and $dA$ is the contact area in the differential segment.
The overall heat transfer coefficient represents the total resistance to heat transfer from one fluid to another. The functional form of $U$ may be derived for any particular geometry by performing a standard conduction analysis on the system of interest. To illustrate this, consider first a planar wall of thickness L, subject to convection on both sides. The overall heat transfer coefficient is inversely proportional to the total resistance $R_{tot}$ to the heat flow. The latter is the sum of (1) resistance $R_{conv,h}$ to convective heat transfer from the hot fluid to the partition between the fluids, (2) resistance $R_p$ to thermal conduction through the partition, and (3) resistance $R_{conv,c}$ to convective heat transfer from the partition to the cold fluid. Therefore for planar wall

$$U = \frac{1}{R_{tot}} = \frac{1}{R_{conv,h} + R_p + R_{conv,c}} = \frac{1}{\frac{1}{h_{conv,h}} + \frac{L}{k} + \frac{1}{h_{conv,c}}}$$

Consider the situation in following picture. Heat is being transferred from the fluid outside (at average temperature of $T_o$). through a dirt or fouling film, through the tube wall, through another fouling film to the inside fluid at a local bulk temperature of $T_i$. $A_i$ and $A_o$ are respectively inside and outside surface areas for heat transfer for a given length of tube. For a plain or bare cylindrical tube




$$\frac{A_o}{A_i} = \frac{2\pi r_o L}{2\pi r_i L} = \frac{r_o}{r_i}$$

The heat transfer rate between the fluid outside the tube and the surface of the outside fouling film is given by an equation of the form $Q/A = h(T_f - T_s)$ where the area is $A_o$ and similarly for the outside convective process where the area is $A_i$. The values of $h_i$ and $h_o$ have to be calculated from appropriate correlations.
On most real heat exchanger surfaces in actual service, a film or deposit of sediment, scale, organic growth, etc. will sooner or later develop. A few fluids such as air are usually clean enough that the fouling is absent or small enough to be neglected. Heat transfer across these films is predominantly by conduction, but the designer seldom knows enough about either the thickness or the thermal conductivity of the film to treat the heat transfer resistance as a conduction problem. Rather, the designer estimates from a table of standard values or from experience a fouling factor $R_f$. $R_f$ is defined in terms of the heat flux Q/A and the temperature difference across the fouling $\Delta T_f$ by the equation:

$$R_f = \frac{\Delta T_f}{Q/A}$$

From this equation, it is clear that $R_f$ is equivalent to a reciprocal heat transfer coefficient for the fouling, $h_f$

$$h_f = \frac{1}{R_f} = \frac{Q/A}{\Delta T_f}$$

and in many books, the fouling is accounted for by a "fouling heat transfer coefficient" which is still an estimated quantity. The effect of including this additional resistance is to provide an exchanger somewhat larger than required when it is clean, so that the exchanger will still provide the desired service after it has been on stream for some time and some fouling has accumulated.
Frequently, but not always the overall heat transfer coefficient is related to outside tube surface:

$$U_o = \frac{1}{\frac{A_o}{h_i A_i} + \frac{R_{fi} A_o}{A_i} + \frac{A_o \ln(r_o / r_i)}{2\pi L k_w} + R_{fo} + \frac{1}{h_o}} \quad [W/m^2K]$$

For thin-walled tubes the overall heat transfer coefficient for planar surface can be used with good accuracy:

$$U_o = \frac{1}{\frac{1}{h_i} + R_{fi} + \frac{r_o - r_i}{k_w} + R_{fo} + \frac{1}{h_o}}$$

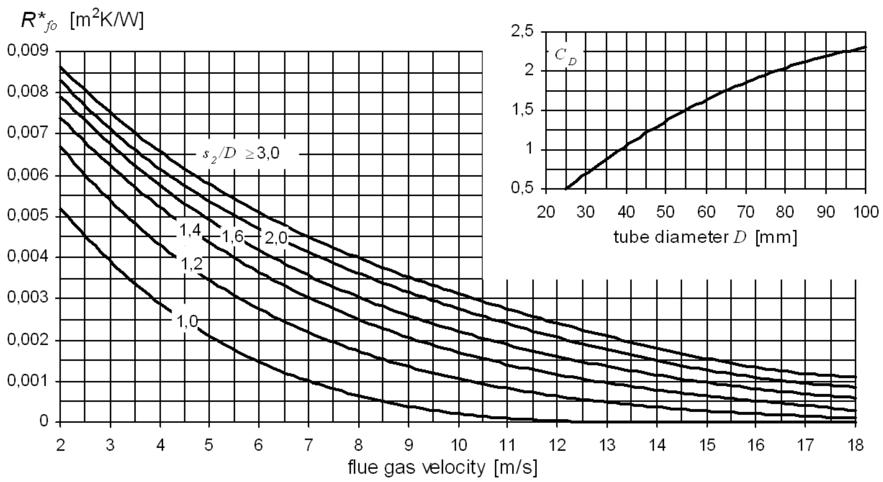
In this equation it is possible to neglect small resistances, e.g. conductive heat transfer through the partition. In the current case, we consider heat conduction in the radial direction of a cylindrical steel tube or heat conduction across a thin steel plate. Value of thermal conductivity of various steel materials depending on composition is shown in the table.

| **Thermal Conductivity of Steel at 20° C (W/mK)** | |
|---|---|
| **Carbon steel** | 35 - 55 |
| **Nickel steel** | 12 - 50 |
| **Chrome steel** | 30 - 60 |
| **Cr - Ni steel** | 16 |
| **Ni - Cr steel** | 14 |
| **Silicon steel** | 31 |
| **Manganese steel** | 38 |
| **Tungsten steel** | 62 |

**FOULING FACTORS (DIRT FACTORS)**

Most process and service fluids will foul the heat-transfer surfaces in an exchanger to a greater or lesser extent. The deposited material will normally have a relatively low thermal conductivity and will reduce the overall coefficient. It is therefore necessary to oversize an exchanger to allow for the reduction in performance during operation. The effect of fouling is allowed for in design by including the inside and outside fouling factors. In tubular boiler heating surfaces working with steam or makeup water the inside fouling is negligible and usually is not taken into account. On the outer side the fouling from flue gas is usually strong depending on used fuel. Flue gas fouling factors quoted as heat-transfer resistances are usually used. They are difficult to predict and are usually based on past experience. For example, value of fouling factor for flue gas from coal burning blowing cross staggered tube bank with row pitch $s_2$ is possible to estimate from following equation with use attached diagram.

$$R_{fo} = C_D \cdot R^*_{fo} + \Delta R$$




Estimating fouling factors introduces a considerable uncertainty into exchanger design; the value assumed for the fouling factor can overwhelm the accuracy of the predicted values of the other coefficients. Fouling factors are often wrongly used as factors of safety in exchanger design.

**Convective heat transfer.** Resistance to the convective heat transfer is inversely proportional to the convective heat transfer coefficient, $h = 1/R_{conv}$. The convective heat transfer coefficient depends on fluid properties, flow geometry, and the flow rate. It is convenient to describe this dependence using several dimensionless numbers, namely the Reynold number

$$Re = \frac{Lv\rho}{\mu},$$

the Prandltl number

$$Pr = \frac{c_p\mu}{k},$$

and the Nusselt number

$$Nu = \frac{hL}{k}.$$

Here, $\rho$, $\mu$, $k$, and $c_p$ are the density, viscosity, thermal conductivity, and heat capacity of the fluid, $v$ is the flow velocity, and $L$ is characteristic length. The choice of $L$ depends on the system geometry. For example, for a flow in a circular pipe, $L$ is the pipe diameter.
The value of the Reynolds number permits us to determine whether the flow is laminar or turbulent. The flow in a commercial circular tube or pipe is usually laminar when the Reynolds number is below 2,300. In the range 2,300 < Re < 4,000 the status of the flow is in transition and for Re > 4,000, flow can be regarded as turbulent. Results for heat transfer in the transition regime are difficult to predict, and it is best to avoid this regime in designing heat exchange equipment. Turbulent flow is inherently unsteady, being characterized by time-dependent fluctuations in the velocity and pressure, but we usually average over these fluctuations and define time-smoothed or time-average velocity and pressure; these time-smoothed entities can be steady or time-dependent (on a time scale much larger than that of the fluctuations), and here we only focus on steady conditions when we discuss either laminar or turbulent flow.
Relationship between *Re*, *Pr*, and *Nu* for examined system is set by theoretical modeling or from results of experimental measurements. A variety of correlations are in use for predicting heat transfer rates. From dimensional analysis, the correlations are usually written in the form

$$Nu = \phi\left(\mathrm{Re}, \mathrm{Pr}, \cdots\right)$$

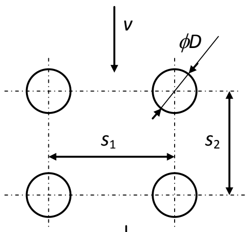
For example, following correlation was defined for a turbulent flow inside a pipe with circular crosssection of diameter $D$

$$Nu = 0.027 Re^{0.8} Pr^{1/3}$$

Application of correlation depends on similarity of

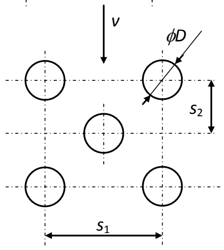
- process (heating, cooling, boiling, condensation, …)
- system geometry
- physical properties
- range of validity.

Following correlations are used for evaluation of convective heat transfer coefficients from flue gas or air to tubular heating surface:




- flow cross in-line tubular bank

$$h_{conv} = 0{,}2 \cdot C_z \cdot C_s \cdot \frac{k}{D} \cdot \left(\frac{v \cdot D \cdot \rho}{\mu}\right)^{0,65} \cdot \mathrm{Pr}^{0,33} \quad [\mathrm{W \cdot m^{-2} \cdot K^{-1}}]$$




- flow cross staggered tubular bank

$$h_{conv} = C_z \cdot C_s \cdot \frac{k}{D} \cdot \left(\frac{v \cdot D \cdot \rho}{\mu}\right)^{0,6} \cdot \mathrm{Pr}^{0,33} \quad [\mathrm{W \cdot m^{-2} \cdot K^{-1}}]$$

  where value of coefficients $C_z$, $C_s$ depends on pitch and number of rows in tube bank

- parallel flow

$$h_{conv} = 0{,}023 \cdot \frac{k}{D} \cdot \left(\frac{v \cdot d_e \cdot \rho}{\mu}\right)^{0,8} \cdot \mathrm{Pr}^{0,4} \cdot C_t \cdot C_l \quad [\mathrm{W \cdot m^{-2} \cdot K^{-1}}]$$

  where value of coefficients $C_t$, $C_l$, depends on temperatures between fluid and wall an on relative length

$d_e$ is equivalent diameter. For tubes and circular ducts $d_e$ = inside diameter. For non-circular ducts

$$d_e = \frac{4 \cdot A}{P} \quad [\mathrm{m}]$$

where $A$ is the cross sectional area and $P$ is the wetted perimeter of the cross-section.

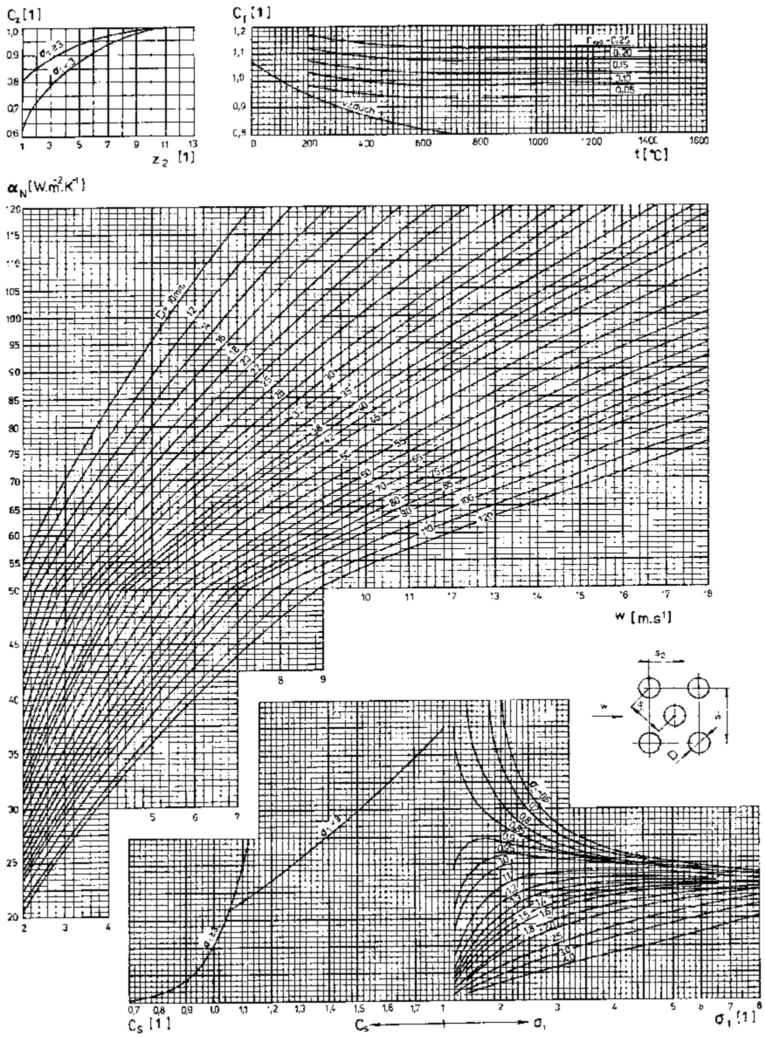
Some correlations are available in form of diagram, e.g. value of convective heat transfer coefficient for flow cross staggered tubular bank is possible to obtain as follows

$$h_{conv} = C_z \cdot C_s \cdot C_f \cdot \alpha_N \quad [\mathrm{W \cdot m^{-2} \cdot K^{-1}}]$$

where

$r_{H2O}$ is volume share of vapor in flue gas
$z_2$ is number of rows in tube bank
and

$$\sigma_1 = \frac{s_1}{D} \qquad \sigma_2 = \frac{s_2}{D}$$




## Finalization of heat exchanger design

Required area of heat exchanger is

$$A = \frac{Q}{U_o \cdot \Delta T_m} \quad [m^2]$$

In some applications, simplified evaluation of overall heat transfer coefficient can be used, e.g. for flue gas water heater (economizer), heat transfer resistances in tube wall and between wall and flowing water are small and can be neglected. Simplified formula for evaluation of overall heat transfer coefficient in economizer is

$$U_{ECO} = \frac{1}{R_{fo} + \frac{1}{h_o}} = \frac{h_o}{1 + R_{fo} h_o}$$

where $h_o$ = ($h_{conv}$ + $h_{rad}$) is the total heat transfer coefficient from flue gas to wall.

Area of tubular heat exchanger consists from parallel tubes. Length of one parallel tube is possible to evaluate from following equation

$$L = \frac{A}{n_t \cdot \pi \cdot D} \quad [m]$$

where $n_t$ is number of parallel tubes depending on fluid flow rate, tube internal diameter and fluid velocity.